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HOLIDAY HOMEWORK-2024

CLASS 12-A

PHYSICS

❖ **Make a small thin notebook for doing the given holiday homework.**

I. Revise the content of all the chapters taught till now.
II. Practice the solved and unsolved numericals of NCERT.
III. Practice more numericals of similar type from other refresher books.
IV. Solve PA1 one question paper in your notebook.
V. Attempt the given questions.

1) What is the nature of symmetry of electric field due to point charge and electric dipole?
2) When an electric dipole is suspended in a uniform electric field, then under what conditions the dipole is in (i) stable equilibrium and (ii) unstable equilibrium.
3) Using Gauss's theorem derive an expression for electric field intensity at a point due to
   (I) A line of charge
   (II) A uniformly charged thin spherical shell
   (III) An infinite plane sheet of charge
   (IV) Two parallel sheets of charge with equal and opposite charge densities.
4) Justify that electrostatic potential is constant throughout the volume of charged conductor and has same value on its surface as inside it.
5) Equipotential surfaces are perpendicular to field lines. Why?
6) A slab of material of dielectric constant K has the same area as the plates offer parallel plate capacitor but has a thickness 3d/4. Find the ratio of the capacitance with dielectric inside it to its capacitance without the dielectric.
7) Derive an expression for the electric potential at any point P at a distance r from the centre of an electric dipole making certain angle with its axis.
8) Explain dielectric polarization, electric susceptibility and dielectric constant. Establish a relation between them.
9) What is drift velocity? Establish a relation between current and drift velocity.

10) State Ohm's law and deduce it from the knowledge of drift velocity of free electrons in a conductor carrying current.
11) Show on a plot, variation of resistivity of (i) a conductor and (ii) a typical semiconductor as a function of temperature.
Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in case of a conductor increases while it decreases in a semiconductor with the rise of temperature.
12) Two cells with internal resistances are connected in parallel. obtain expressions for the equivalent internal resistance and emf of the combination.

Mathematics

Relations and Functions

1. Let the function 'f' : $N \to N$ be defined by $f(x) = 2x + 3$, $\forall\, x \in N$. Then 'f' is **(1 mark)**
   (*a*) not onto
   (*b*) bijective function
   (*c*) many-one, into function
   (*d*) None of these
2. A relation defined in a non-empty set A, having n elements, has **(1 mark)**
   (*a*) n relations (*b*) 2 relations
   (*c*) n^2 relations (*d*) 2^{n^2} relations
3. A relation R in human beings defined as $R = \{(a, b) : a, b \in$ human beings ; a loves $b\}$ is **(1 mark)**
   (*a*) reflexive
   (*b*) symmetric and transitive
   (*c*) equivalence
   (*d*) neither of these
4. A function $f : R \to R$ is defined as $f(x) = x^3 + 1$. Then the function has **[CBSE 2021] (1 mark)**
   (*a*) no minimum value
   (*b*) no maximum value
   (*c*) both maximum and minimum values
   (*d*) neither maximum value nor minimum value
5. Let $A = \{a, b, c\}$ and the relation R be defined on A, as follows: $R = \{(a, a), (b, c), (a, b)\}$. Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive. **[NCERT Exemplar] (2 marks)**
6. Given set $A = \{a, b, c\}$. Is relation $R = \{(a, c)\}$ transitive? **(2 marks)**
7. Prove that the Greatest Integer Function $f : R \to R$, given by $f(x) = [x]$ is neither one-one nor onto. Where $[x]$ denotes the greatest integer less than or equal to x. **[NCERT] (2 marks)**
8. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one. **[NCERT] (2 marks)**
9. Let $f : N \to N$ be defined by $f(x) = 3x$. Show that f is not onto function. **[HOTS] (2 marks)**
10. Let the function $f : R \to R$ be defined by $f(x) = \cos x$ $\forall\, x \in R$. Show that f is neither one-one nor onto. **[NCERT Exemplar] (2 marks)**
11. Determine whether the relation R defined on the set R of all real numbers as $R = \{(a, b) : a, b \in R$ and $a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive. **[Ajmer 2015] (3 marks)**
12. Given a non empty set X, consider $P(X)$ which is the set of all subsets of X. Define the relation R in $P(X)$ as follows: For subsets A, B in $P(X)$, ARB if and only if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify your answer. **[NCERT] (3 marks)**
13. Let A and B be sets. Show that $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is bijective function. **[NCERT] (3 marks)**
14. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$. **[NCERT] (5 marks)**
15. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1. Also, write the equivalence class [2]. **[CBSE 2018] (5 marks)**
16. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$ if $ad\,(b + c) = bc\,(a + d)$. Show that R is an equivalence relation. **[DoE; Delhi 2015] (5 marks)**
17. Let $f : N \to N$ be defined by
$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in N.$$
State whether the function f is bijective. Justify your answer. **[NCERT] (5 marks)**
18. Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function. **[NCERT] (5 marks)**

Relations and Functions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A):** In set $A = \{1, 2, 3\}$ a relation R defined as $R = \{(1, 1), (2, 2)\}$ is reflexive.

   Reason (R): A relation R is reflexive in set A if $(a, a) \in R$ for all $a \in A$.

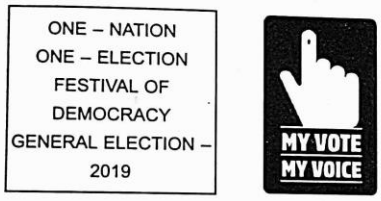
2. **Assertion (A):** In set $A = \{a, b, c\}$ relation R in set A, given as $R = \{(a, c)\}$ is transitive.

   Reason (R): A singleton relation is transitive.

3. **Assertion (A):** Given set $A = \{1, 2, 3, \ldots 9\}$ and relation R in set $A \times A$ defined by $(a, b)\ R\ (c, d)$ if $a + d = b + c$, be an equivalence relation. The ordered pair $(1, 3)$ belongs to equivalence class related to $[(5, 3)]$

   Reason (R): Any ordered pair of $A \times A$ belongs to equivalence class $[(5, 3)]$ if $(x, y)R\ (5, 3)\ \forall\ (x, y) \in A \times A$.

1. A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever



   Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

   $\{R = \{(V_1, V_2): V_1, V_2 \in I$ and both use their voting right in general election – 2019$\}$

   (*i*) Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election-2019. Check whether X is related to Y or not.

   (*ii*) Mr. 'X' and his wife 'W' both exercised their voting right in general election-2019. Show that $(X, W) \in R$ and $(W, X) \in R$.

   (*iii*) Three friends F_1, F_2 and F_3 exercised their voting right in general election-2019. Show that

   $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$.

   OR

   Show that the relation R defined on set I is an equivalence relation.

2. Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted that possible outcomes of the throw every time belongs to set $\{1, 2, 3, 4, 5, 6\}$. Let A be the set of players while B be the set of all possible outcomes.

   $A = \{S, D\}$, $B = \{1, 2, 3, 4, 5, 6\}$

   (*i*) Let $R : B \to B$ be defined by $R = \{(x, y) : y$ is divisible by $x\}$. Show that relation R is reflexive and transitive but not symmetric.

   (*ii*) Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then check whether R is an equivalence relation.

   (*iii*) Raji wants to know the number of functions from A to B. How many number of functions are possible?

   OR

   Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

Inverse trigonometric functions

1. Principal value of the expression $\cos^{-1}[\cos(-680°)]$ is
   (a) $\frac{2\pi}{9}$ (b) $-\frac{2\pi}{9}$
   (c) $\frac{34\pi}{9}$ (d) $\frac{\pi}{9}$

2. If $\tan^{-1} x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, then x is equal to
   (a) $\frac{5\pi}{4}$ (b) 1
   (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

3. Find the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)$.

Write the principal value of each of the following (Exercises 4 to 9):

4. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ [NCERT Exemplar; DoE]

5. $\sec^{-1}(-2)$ [DoE]

6. $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$. [Delhi 2012]

7. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

8. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

9. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ [NCERT]

10. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ [NCERT]

11. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ [NCERT]

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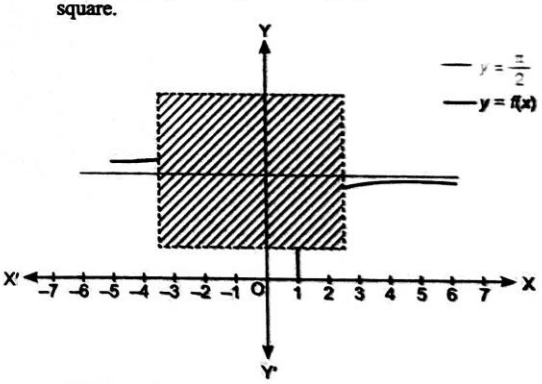
1. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is (1 Mark)
   (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$
   (c) $-\frac{\pi}{6}$ (d) $\frac{\pi}{6}$

2. The value of $\cos^{-1}\left(\frac{1}{2}\right) + 3\sin^{-1}\left(\frac{1}{2}\right)$ is equal to (1 Mark)
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$
   (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

3. The greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ are respectively (1 Mark)
   (a) $\frac{\pi^2}{8}, \frac{5\pi^2}{4}$ (b) $\frac{\pi^2}{4}, \frac{5\pi^2}{8}$
   (c) $\frac{5\pi^2}{4}, \frac{\pi^2}{8}$ (d) $\frac{5\pi^2}{8}, \frac{\pi^2}{4}$

4. The value of $\sin\frac{3\pi}{2} - \sin(\sec^{-1} t + \operatorname{cosec}^{-1} t)$, when $|t| \geq 1$. [CBSE Learning Framework] (1 Mark)
   (a) 0 (b) –1
   (c) 1 (d) –2

5. Shown below is graph of function 'f' whose domain is $R - (-1, 1)$ some portion of graph is hidden behind square.



   Which of the following is '$f(x)$'? (1 Mark)
   (a) $\tan^{-1}x$ (b) $\cot^{-1}x$
   (c) $\sec^{-1}x$ (d) $\operatorname{cosec}^{-1}x$

6. $\cot^{-1}x = \cos^{-1}(-1) - \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$
   Based on above find $\tan^{-1}\left(\frac{1}{x}\right)$ using the principal value of inverse trigonometric function. Show your work. (2 Marks)

7. Find the domain of the function:
$f(x) = \frac{1}{2}\sec^{-1}(5x - 3)$ **(2 Marks)**

8. Find the range of principal value branch of the function:
$f(x) = 3\cos^{-1}\left(\frac{1}{2x-1}\right) - 2$. Show your work. **(2 Marks)**

9. Find the principal value of $\text{cosec}^{-1}(2)$. [NCERT] **(2 Marks)**

10. Evaluate $\tan^{-1}\left\{\sin\left(-\frac{\pi}{2}\right)\right\}$. [NCERT Exemplar] **(2 Marks)**

11. Write the value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.
[Foreign 2014] **(2 Marks)**

12. Write one branch of $\sin^{-1}x$ other than the principal branch. **(2 Marks)**

13. Find the principal value of $\tan^{-1}(-1)$ [NCERT] **(2 Marks)**

14. Find the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.
[NCERT; HOTS] **(2 Marks)**

15. Find the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$. [Foreign 2013] **(2 Marks)**

16. Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$.
[NCERT Exemplar; Foreign 2013] **(2 Marks)**

17. Write the principal value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.
[NCERT; HOTS] **(2 Marks)**

18. Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.
[CBSE 2020] **(2 Marks)**

19. Find the principal value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$. [DoE] **(2 Marks)**

ASSERTION AND REASON QUESTIONS

In the following questions, a statement of assertion (*A*) is followed by a statement of reason (*R*). Choose the correct answer out of the following choices.

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.

(*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.

(*c*) *A* is true but *R* is false.

(*d*) *A* is false but *R* is true.

1. Assertion (*A*): Inverse of sine function exists in interval $[0, \pi]$

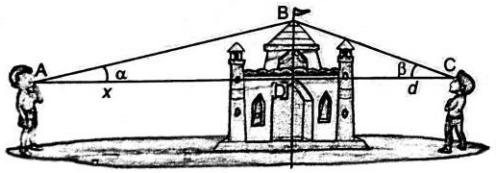
Reason (*R*): $\sin^{-1}$ function becomes bijective if we restrict its domain to $[-1, 1]$.

2. Assertion (*A*): Principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Reason (*R*): The range of principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\tan(-x) = -\tan x$.

CASE-BASED QUESTIONS

1.



Two men on either side of a temple of 30 metres high from the level of eye observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ metres. Based on the above information answer the following:

(*i*) Find $\angle CAB$ and $\angle ACB$ (*ii*) Find $\angle ABC$

(*iii*) Find the principal value of $\sin^{-1}\left\{\sin\left(\alpha + \frac{2\pi}{3}\right)\right\}$

OR

Find the principal value of $\cos^{-1}\left\{\cos\left(\beta + \frac{\pi}{3}\right)\right\}$

2. Let $f: A \to B$ be a bijective function then $f^{-1}: B \to A$ is a function such that $f[f^{-1}(x)] = x \ \forall \ x \in B$ and $f^{-1}[f(x)] = x \ \forall \ x \in A$. For ensuring bijectivity, domain of trigonometric function are restricted.

Based on the above information, answer the following questions:

(*i*) Find the value of $\sin^{-1}(\sin 10)$.

(*ii*) Find the value of $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$ if $x \in (-\pi, \pi)$.

(*iii*) Find the value of function $\sin(\tan^{-1} x)$.

OR

Find the value of function $\cos[\text{cosec}^{-1}(x)]$.

Inverse trigonometric functions

1. Solution of $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is

   (a) $-\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

   (c) $\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

2. If $\alpha \le 2\sin^{-1}x + \cos^{-1}x \le \beta$, then α and β are

   (a) $\alpha = 0, \beta = \pi$ (b) $\alpha = \pi, \beta = 0$

   (c) $\alpha = -\frac{\pi}{2}, \beta = \frac{\pi}{2}$ (d) $\alpha = \frac{\pi}{2}, \beta = -\frac{\pi}{2}$

3. If $\tan^{-1}x = \frac{\pi}{10}$ for some $x \in R$, then the value of $\cot^{-1}x$ is

   (a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) $\frac{4\pi}{5}$

4. The principal value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ is

   (a) $\frac{\pi}{8}$ (b) $\frac{3\pi}{8}$ (c) $-\frac{\pi}{8}$ (d) $-\frac{3\pi}{8}$

   [CBSE 2021]

5. The principal value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

   (a) $\frac{\pi}{12}$ (b) π (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

   [CBSE 2021]

6. Show that $\sin^{-1}\left(\sqrt{\frac{a-x}{2a}}\right) = \frac{1}{2}\cos^{-1}\frac{x}{a}$.

Write the principal values in Exercises 7 to 10:

7. $\text{cosec}^{-1}(2)$ 8. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

9. $\tan^{-1}(-\sqrt{3})$ 10. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Write the value in Exercises 11 to 13:

11. $\text{cosec}^{-1}(\sqrt{2}) + \sec^{-1}(\sqrt{2})$

12. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\cos\frac{2\pi}{3}\right)$

13. $\tan^{-1}(\sqrt{3}) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

14. What is the domain of the function $\text{cosec}^{-1}x$?

15. Write one branch of $\tan^{-1}x$ other than the principal branch.

Evaluate in Exercises 16 to 27:

16. $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{3}{2}\right)\right\}$ 17. $\text{cosec}^{-1}\left\{\text{cosec}\left(-\frac{\pi}{4}\right)\right\}$

18. $\cos\left\{\frac{\pi}{3} - \cos^{-1}\left(\frac{1}{2}\right)\right\}$ 19. $\sec^2(\tan^{-1}2)$

20. $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$

21. $\tan^{-1}\{\cos\pi\}$

22. Find the value of $\sin\left\{2\cot^{-1}\left(-\frac{5}{12}\right)\right\}$.

In the following questions, a statement of assertion (*A*) is followed by a statement of reason (*R*). Choose the correct answer out of the following choices.

(a) Both *A* and *R* are true and R is the correct explanation of *A*.

(b) Both *A* and *R* are true but *R* is not the correct explanation of *A*.

(c) *A* is true but *R* is false.

(d) *A* is false but *R* is true.

23. **Assertion (*A*):** The domain of the function $f(x) = \cos^{-1}(3x + 1)$ is $\left[\frac{-2}{3}, 0\right]$

    Reason (*R*): Domain of $\cos^{-1}$ is $[-1, 1]$

24. **Assertion (*A*):** $\sin^{-1}(-1.0001)$ is defined.

    Reason (*R*): Domain of $\sin^{-1}x$ is $[-1, 1]$.

25. **Assertion (*A*):** Principal value of $\sin^{-1}\left(\sin\frac{17\pi}{18}\right)$ is $\frac{\pi}{18}$.

    Reason (*R*): Domain of principal value branch of $\sin^{-1}$ is $[-1, 1]$.

26. **Assertion (*A*):** The range of principal value branch of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

    Reason (*R*): Domain of principal value branch of $\text{cosec}^{-1}x$ is $[-1, 1] - \{0\}$

27. A teacher gives a table of the domain and range of inverse trigonometric functions to the students and told them that when we dealing with the inverse trigonometric functions, we need to careful about their range, which is defined from restricted domain of trigonometric functions.

Matrix

1. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A'$ then

   (a) $x = 0, y = 5$ (b) $x = y$

   (c) $x + y = 5$ (d) $x - y = 5$

2. If a matrix A is both symmetric and skew symmetric then matrix A is

   (a) a scalar matrix

   (b) a diagonal matrix

   (c) a zero matrix of order $n \times n$

   (d) a rectangular matrix.

3. If A is a skew symmetric matrix then A^2 is a

   (a) square matrix

   (b) diagonal matrix

   (c) symmetric matrix

   (d) skew symmetric matrix

4. If A and B are two matrices such that their multiplication is defined, then $(AB)'$

   (a) $A'B'$ (b) AB

   (c) AB' (d) $B'A'$

5. For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is [CBSE 2021]

   (a) $2I$ (b) $3I$ (c) I (d) $5I$

6. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $P - Q$ is [CBSE 2021]

   (a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

   (c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

7. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$. [NCERT]

8. Is matrix $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ symmetric or skew symmetric? Give a reason.

9. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew symmetric matrix, where A^T denotes the transpose of A.

10. If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that $A + A^T$ is a symmetric matrix, where A^T denotes the transpose of matrix A.

11. For the matrix A, show that $A + A^T$ is a symmetric matrix. [HOTS]

12. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b. [Delhi 2016]

13. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $B = [-2 \ -1 \ -4]$, verify that $(AB)' = B'A'$.

14. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, verify that $AA' = I$

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1. If $F(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then $F(x)\,F(y)$ is equal to **(1 Mark)**

   (a) $F(x)$ (b) $F(xy)$

   (c) $F(x + y)$ (d) $F(x - y)$

2. The matrix A satisfies the equation $\begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then matrix A is **(1 Mark)**

   (a) $\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$

3. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then A^6 is equal to **(1 Mark)**

   (a) zero matrix (b) A

   (c) I (d) None of these

4. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 - 5A - 7I$ is **(1 Mark)**

(a) a zero matrix (b) an identity matrix

(c) diagonal matrix (d) None of these

5. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

[NCERT Exemplar] **(2 Marks)**

6. Construct $a_{2\times 2}$ matrix where, $a_{ij} = |-2i + 3j|$.

[NCERT Exemplar] **(2 Marks)**

7. If $A = [a_{ij}] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = [b_{ij}] = [-3\ 2]$, then find $\frac{a_{11} \cdot b_{12}}{a_{21} \cdot a_{12}} + b_{11}$. **(2 Marks)**

8. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A. [Delhi 2013] **(2 Marks)**

9. Find the value of $x + y$ from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

[NCERT; Bhubaneshwar 2015; AI 2012] **(2 Marks)**

10. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then write the value of $(x + y)$. [Delhi 2013(C)] **(2 Marks)**

11. Simplify

$$\tan\theta\begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix} + \sec\theta\begin{bmatrix} -\tan\theta & -\sec\theta \\ -\sec\theta & \tan\theta \end{bmatrix}.$$

(2 Marks)

12. Solve the following matrix equation for x,

$$[x\ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O.$$ [Delhi 2014] **(2 Marks)**

13. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, then find $(3A - B)$.

[Guwahati 2015] **(2 Marks)**

14. Write the element a_{12} of the matrix $A = [a_{ij}]_{2\times 2}$, whose elements a_{ij} are given by $a_{ij} = e^{2ix} \sin jx$.

[Punchkula 2015] **(2 Marks)**

15. Write a 3×3 skew symmetric matrix.

[Chennai 2015] **(2 Marks)**

16. If $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 5 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{bmatrix}$, find B. **(2 Marks)**

17. Find x, if $[x\ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = O$. [HOTS] **(2 Marks)**

18. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 16I$.

[Patna 2015] **(3 Marks)**

19. If $f(x) = x^2 - 4x + 1$, find $f(A)$, when $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. **(3 Marks)**

20. Find the matrix X such that,

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$$ [HOTS] **(3 Marks)**

21. If A is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$. [NCERT Exemplar; AI 2014] **(3 Marks)**

22. Find x, if $[x\ -5\ -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$.

[NCERT] **(3 Marks)**

23. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

[NCERT] **(5 Marks)**

24. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$.

[Delhi 2015] **(5 Marks)**

25. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 4I = O$. Hence find A^{-1}.

[Guwahati 2015] **(5 Marks)**

Determinants

1. A and B are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If $|A| = 2$, then $|B|$ is

   (a) 16 (b) 4 (c) 6 (d) $\frac{1}{16}$.

2. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then adj A is

   (a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (b) $\begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

   (c) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (d) $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

3. The matrix $\begin{bmatrix} 4+3k & 3 \\ 1+2k & 2 \end{bmatrix}$ is singular matrix, for k equal to

   (a) 0 (b) –1

   (c) 1 (d) no value of k

4. If the value of a third order determinant is 7, then the value of a determinant formed by replacing each element by its cofactor will be

   (a) 7 (b) –7

   (c) 49 (d) 14

5. The values of 'x' for which $\begin{vmatrix} 6 & -2 \\ 2 & 4 \end{vmatrix} = x^2 - 12x$ are

   (a) –2, 14 (b) 2, –14

   (c) –2, –14 (d) None of these

6. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

7. Determinant of a non-singular matrix P of order 2 is 12. Find the determinant of P^{-1}.

8. If $A^2 - 3A + I = O$ and A is a non-singular matrix, then write A^{-1} in terms of I and A.

9. If the value of third order determinant is 12, then find the value of the determinant formed by its cofactors.

10. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, find the value of x.

11. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}. [CBSE 2020]

12. Find A^{-1}, if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that $A^{-1} = \frac{A^2 - 3I}{2}$. [NCERT Exemplar]

13. Find the adjoint of matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$ and verify that

    $A(\text{adj } A) = (\text{adj } A)A = |A|I$. [NCERT]

14. Find the inverse of matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$. [NCERT]

Determinants

1. Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations $x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 2$

2. Using matrix method solve the following system of linear equations: $x - y + 2z = 7$; $3x + 4y - 5z = -5$, $2x - y + 3z = 12$ **[NCERT; Delhi 2012]**

3. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is ₹ 70. Find cost of each item per kg by matrix method. **[NCERT]**

4. If $A = \begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Hence using A^{-1} solve the system of equations $2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$; $x + y - 2z = -3$. **[CBSE 2020; AI 2017]**

5. If $A = \begin{bmatrix}1 & 2 & 5\\ 1 & -1 & -1\\ 2 & 3 & -1\end{bmatrix}$ find A^{-1} and hence solve the system of equations $x + 2y + 5z = 10$; $x - y - z = -2$; and $2x + 3y - z = -11$. **[Foreign 2017]**

6. If $A = \begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$, find A^{-1} and hence solve the system of equations $x - 2y = 10$; $2x + y + 3z = 8$; and $-2y + z = 7$. **[Foreign 2017]**

☆ Do Holiday HW in a separate notebook.

1. Maximum value of $\Delta = \begin{vmatrix}1 & 1 & 1+\cos\theta\\ 1 & 1+\sin\theta & 1\\ 1 & 1 & 1\end{vmatrix}$, θ is a real number is **(1 Mark)**

   (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

2. If A and B are invertible matrices then which of the following is not correct **(1 Mark)**

   (a) $AdjA = |A|.A^{-1}$ (b) $\det(A^{-1}) = (\det A)^{-1}$

   (c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A + B)^{-1} = A^{-1} + B^{-1}$

3. Let A be a non-angular square matrix of order 3 × 3, then $|A \cdot \text{adj } A|$ is equal to **(1 Mark)**

   (a) $|A|^3$ (b) $|A|^2$ (c) $|A|$ (d) $3|A|$

4. Let A be a square matrix of order 3 × 3 and k a scalar, then $|kA|$ is equal to **(1 Mark)**

   (a) $k|A|$ (b) $|k||A|$

   (c) $k^3|A|$ (d) None of these

5. If C_{ij} denotes the cofactor of element p_{ij} of the matrix $P = \begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & 2 & 4\end{bmatrix}$, then the value of $C_{31} \cdot C_{23}$ is **(1 Mark)**

   (a) 5 (b) 24 (c) −24 (d) −5

6. If for the matrix $A = \begin{bmatrix}\alpha & -2\\ -2 & \alpha\end{bmatrix}$, $|A^3| = 125$, then the value of α is **(1 Mark)**

   (a) ± 3 (b) −3 (c) ± 1 (d) 1

7. If A is a square matrix of order 3 and $|A| = -5$, then $|adj\ A|$ is **(1 Mark)**

   (a) 125 (b) −25 (c) 25 (d) ±25

8. Let matrix $X = [x_{ij}]$ is given by $X = \begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}$. Then the matrix $Y = [m_{ij}]$, where m_{ij} = Minor of x_{ij}, is **(1 Mark)**

   (a) $\begin{bmatrix}7 & -5 & -3\\ 19 & 1 & -11\\ -11 & 1 & 7\end{bmatrix}$ (b) $\begin{bmatrix}7 & -19 & -11\\ 5 & -1 & -1\\ 3 & 11 & 7\end{bmatrix}$

   (c) $\begin{bmatrix}7 & 19 & -11\\ -3 & 11 & 7\\ -5 & -1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}7 & 19 & -11\\ -1 & -1 & 1\\ -3 & -11 & 7\end{bmatrix}$

9. If $x = -4$ is a root of $\begin{vmatrix}x & 2 & 3\\ 1 & x & 1\\ 3 & 2 & x\end{vmatrix} = 0$, then the sum of the other two roots is **(1 Mark)**

   (a) 4 (b) −3 (c) 2 (d) 5

10. If points (2, 0), (0, 5) and (x, y) are collinear, then show that $\frac{x}{2} + \frac{y}{5} = 1$. **(2 Marks)**

11. If A is a square matrix of order 3 and $|3A| = k|A|$, then write the value of k. **(2 Marks)**

12. A is a non-singular matrix of order 3 and $|A| = -4$. Find $|\text{adj } A|$. **[HOTS] (2 Marks)**