12th Mathematics Holiday Home Work

RELATIONS & FUNCTION

SECTION A

1. Show that the relation R defined by $R = \{(a, b) : a - b \text{ is divisible by } 3; a, b \in Z\}$ is an equivalence relation. [CBSE 2008]
2. Show that the relation R on the set Z of integers, given by $R = \{(a, b) : 2 \text{ divides } a - b\}$, is an equivalence relation. [NCERT]
3. Prove that the relation R on Z defined by $(a, b) \in R \Leftrightarrow a - b$ is divisible by 5 is an equivalence relation on Z. [CBSE 2010]
4. Let n be a fixed positive integer. Define a relation R on Z as follows:
   $(a, b) \in R \Leftrightarrow a - b$ is divisible by n.
   Show that R is an equivalence relation on Z.
5. Let Z be the set of integers. Show that the relation $R = \{(a, b) : a, b \in Z \text{ and } a + b \text{ is even}\}$ is an equivalence relation on Z.
6. m is said to be related to n if m and n are integers and $m - n$ is divisible by 13. Does this define an equivalence relation?
7. Let R be a relation on the set A of ordered pairs of non-zero integers defined by $(x, y)\ R\ (u, v)$ iff $xv = yu$. Show that R is an equivalence relation. [NCERT]
8. Show that the relation R on the set $A = \{x \in Z ; 0 \le x \le 12\}$, given by $R = \{(a, b) : a = b\}$, is an equivalence relation. Find the set of all elements related to 1.
9. Let L be the set of all lines in XY-plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.
10. Show that the relation R, defined on the set A of all polygons as
    $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$,
    is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5? [NCERT]
11. Let R be the relation defined on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even}\}$. Show that R is an equivalence relation. Further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$. [NCERT]
12. Check whether the relation R on the set N of natural numbers given by $R = \{(a, b) : a$ is divisor of $b\}$ is reflexive, symmetric or transitive. Also, determine whether R is an equivalence relation. [CBSE 2020]

SECTION B

1. Let $A = \{-1, 0, 1\}$ and $f = \{(x, x^2) : x \in A\}$. Show that $f : A \to A$ is neither one-one nor onto.

2. Classify the following functions as injection, surjection or bijection:

(i) $f : N \to N$ given by $f(x) = x^2$ (ii) $f : Z \to Z$ given by $f(x) = x^2$

(iii) $f : N \to N$ given by $f(x) = x^3$ (iv) $f : Z \to Z$ given by $f(x) = x^3$

(v) $f : R \to R$, defined by $f(x) = |x|$ (vi) $f : Z \to Z$, defined by $f(x) = x^2 + x$

(vii) $f : Z \to Z$, defined by $f(x) = x - 5$ (viii) $f : R \to R$, defined by $f(x) = \sin x$

(ix) $f : R \to R$, defined by $f(x) = x^3 + 1$ (x) $f : R \to R$, defined by $f(x) = x^3 - x$

(xi) $f : R \to R$, defined by $f(x) = \sin^2 x + \cos^2 x$

(xii) $f : Q - \{3\} \to Q$, defined by $f(x) = \dfrac{2x + 3}{x - 3}$

(xiii) $f : Q \to Q$, defined by $f(x) = x^3 + 1$ (xiv) $f : R \to R$, defined by $f(x) = 5x^3 + 4$

(xv) $f : R \to R$, defined by $f(x) = 3 - 4x$ (xvi) $f : R \to R$, defined by $f(x) = 1 + x^2$

(xvii) $f : R \to R$, defined by $f(x) = \dfrac{x}{x^2 + 1}$ [CBSE 2018, NCERT EXEMPLAR]

3. Let $A = [-1, 1]$. Then, discuss whether the following functions from A to itself are one-one, onto or bijective:

(i) $f(x) = \dfrac{x}{2}$ (ii) $g(x) = |x|$ (iii) $h(x) = x^2$ [NCERT EXEMPLAR]

4. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective:

(i) $\{(x, y) : x$ is a person, y is the mother of $x\}$ (ii) $\{(a, b) : a$ is a person, b is an ancestor of $a\}$

[NCERT EXEMPLAR]

INVERSE TRIGONOMETRIC FUNCTIONS

SECTION A

1. **Find the principal values of each of the following:**

(i) $\cot^{-1}(-\sqrt{3})$ **(ii)** $\cot^{-1}(\sqrt{3})$ **(iii)** $\cot^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$ **(iv)** $\cot^{-1}\left(\tan\dfrac{3\pi}{4}\right)$

2. **Find the domain of** $f(x) = \cot x + \cot^{-1} x$.

3. **Evaluate each of the following:**

(i) $\cot^{-1}\dfrac{1}{\sqrt{3}} - \operatorname{cosec}^{-1}(-2) + \sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$ **(ii)** $\cot^{-1}\left\{2\cos\left(\sin^{-1}\dfrac{\sqrt{3}}{2}\right)\right\}$

(iii) $\operatorname{cosec}^{-1}\left(-\dfrac{2}{\sqrt{3}}\right) + 2\cot^{-1}(-1)$

(iv) $\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\dfrac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\dfrac{\pi}{2}\right)\right)$ [NCERT EXEMPLAR

ANSWERS

SECTION B

1. Write each of the following in the simplest form:

(i) $\cot^{-1}\left\{\frac{a}{\sqrt{x^2-a^2}}\right\}, |x|>a$

(ii) $\tan^{-1}\left\{x+\sqrt{1+x^2}\right\}, x\in R$

(iii) $\tan^{-1}\left\{\sqrt{1+x^2}-x\right\}, x\in R$

(iv) $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}, x\neq 0$

(v) $\tan^{-1}\left\{\frac{\sqrt{1+x^2}+1}{x}\right\}, x\neq 0$

(vi) $\tan^{-1}\sqrt{\frac{a-x}{a+x}}, -a<x<a$

(vii) $\tan^{-1}\left\{\frac{x}{a+\sqrt{a^2-x^2}}\right\}, -a<x<a$

(viii) $\sin^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}, -\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

(ix) $\sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}, 0<x<1$

(x) $\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

SECTION C

1. **Evaluate :**

(i) $\cos\left\{\sin^{-1}\left(-\frac{7}{25}\right)\right\}$ (ii) $\sec\left\{\cot^{-1}\left(-\frac{5}{12}\right)\right\}$ (iii) $\cot\left\{\sec^{-1}\left(-\frac{13}{5}\right)\right\}$

2. **Evaluate :**

(i) $\tan\left\{\cos^{-1}\left(-\frac{7}{25}\right)\right\}$ (ii) $\text{cosec}\left\{\cot^{-1}\left(-\frac{12}{5}\right)\right\}$ (iii) $\cos\left\{\tan^{-1}\left(-\frac{3}{4}\right)\right\}$

3. **Evaluate :** $\sin\left\{\cos^{-1}\left(-\frac{3}{5}\right)+\cot^{-1}\left(-\frac{5}{12}\right)\right\}$.

SECTION D

1. If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{3}$ and $\cos^{-1}x-\cos^{-1}y=\frac{\pi}{6}$, find the values of x and y.

2. If $\cot\left(\cos^{-1}\frac{3}{5}+\sin^{-1}x\right)=0$, find the values of x.

3. If $(\sin^{-1}x)^2+(\cos^{-1}x)^2=\frac{17\pi^2}{36}$, find x.

4. Solve : $\sin\left\{\sin^{-1}\frac{1}{5}+\cos^{-1}x\right\}=1$ **[CBSE 2**

5. Solve the following equations:

(i) $\sin^{-1}x=\frac{\pi}{6}+\cos^{-1}x$ (ii) $4\sin^{-1}x=\pi-\cos^{-1}x$

(iii) $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$ (iv) $5\tan^{-1}x+3\cot^{-1}x=2\pi$

SECTION E

1. Evaluate the following:

(i) $\tan\left\{2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right\}$ (ii) $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$ [CBSE 2013, NCERT EXEMPLAR]

(iii) $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ (iv) $\sin\left(2\tan^{-1}\frac{2}{3}\right)+\cos\left(\tan^{-1}\sqrt{3}\right)$ [NCERT EXEMPLAR]

2. Prove the following results:

(i) $2\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{24}{7}$ (ii) $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}=\frac{\pi}{4}$ [NCERT EXEMPLAR]

(iii) $\tan^{-1}\frac{2}{3}=\frac{1}{2}\tan^{-1}\frac{12}{5}$ (iv) $\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{4}$ [CBSE 2010]

(v) $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$ (vi) $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

(vii) $2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\tan^{-1}\frac{4}{7}$ [NCERT]

(viii) $2\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$ [CBSE 2011]

(ix) $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$ [CBSE 2011, 2020]

(x) $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}=\frac{1}{2}\sin^{-1}\frac{4}{5}$ [CBSE 2010 C]

3. Find the values of each of the following:

(i) $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$ (ii) $\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right), |x|\geq 1$

4. If $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=\tan^{-1}\frac{2x}{1-x^2}$, then prove that $x=\frac{a-b}{1+ab}$.

MATRIX

SECTION A

1. If A is an $m\times n$ matrix and B is $n\times p$ matrix does AB exist? If yes, write its order
2. If $A=\begin{bmatrix}2&1&4\\4&1&5\end{bmatrix}$ and $B=\begin{bmatrix}3&-1\\2&2\\1&3\end{bmatrix}$. Write the orders of AB and BA.
3. If $A=\begin{bmatrix}4&3\\1&2\end{bmatrix}$ and $B=\begin{bmatrix}-4\\3\end{bmatrix}$, write AB.
4. If $A=\begin{bmatrix}1\\2\\3\end{bmatrix}$, write AA^T.
5. Give an example of two non-zero 2×2 matrices A and B such that $AB=O$.
6. If $A=\begin{bmatrix}2&3\\5&7\end{bmatrix}$, find $A+A^T$.
7. If $A=\begin{bmatrix}i&0\\0&i\end{bmatrix}$, write A^2.
8. If $A=\begin{bmatrix}\cos x&\sin x\\-\sin x&\cos x\end{bmatrix}$, find x satisfying $0<x<\frac{\pi}{2}$ when $A+A^T=I$
9. If $A=\begin{bmatrix}\cos x&-\sin x\\\sin x&\cos x\end{bmatrix}$, find AA^T
10. If $\begin{bmatrix}1&0\\y&5\end{bmatrix}+2\begin{bmatrix}x&0\\1&-2\end{bmatrix}=I$, where I is 2×2 unit matrix. Find x and y.

SECTION B

11. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, satisfies the matrix equation $A^2 = kA$, write the value of k.
12. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, then write the value of λ.
13. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^2.
14. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^3.
15. If $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$, find A^4.
16. If $\begin{bmatrix} x & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$, find x
17. If $A = [a_{ij}]$ is a 2×2 matrix such that $a_{ij} = i + 2j$, write A. [CBSE 2008]
18. Write matrix A satisfying $A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$.
19. If $A = [a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew-symmetric.
20. For any square matrix write whether AA^T is symmetric or skew-symmetric.
21. If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i a_{ii}$.
22. If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i \sum_j a_{ij}$.
23. If A and B are symmetric matrices, then write the condition for which AB is also symmetric.
24. If B is a skew-symmetric matrix, write whether the matrix $AB\,A^T$ is symmetric or skew-symmetric.
25. If B is a symmetric matrix, write whether the matrix $AB\,A^T$ is symmetric or skew-symmetric.
26. If A is a skew-symmetric and $n \in N$ such that $(A^n)^T = \lambda A^n$, write the value of λ.
27. If A is a symmetric matrix and $n \in N$, write whether A^n is symmetric or skew-symmetric or neither of these two.
28. If A is a skew-symmetric matrix and n is an even natural number, write whether A^n is symmetric or skew-symmetric or neither of these two.
29. If A is a skew-symmetric matrix and n is an odd natural number, write whether A^n is symmetric or skew-symmetric or neither of the two.
30. If A and B are symmetric matrices of the same order, write whether $AB - BA$ is symmetric or skew-symmetric or neither of the two.
31. Write a square matrix which is both symmetric as well as skew-symmetric.
32. Find the values of x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$. [CBSE 2008, 2019]
33. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y. [CBSE 2008]
34. Find the value of x from the following: $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$. [CBSE 2009]

35. Find the value of y, if $\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$. [CBSE 2010]

36. Find the value of x, if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$. [CBSE 2010]

37. If matrix $A = [1 \ \ 2 \ \ 3]$, write AA^T. [CBSE 2010]

38. If $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$, then find x. [CBSE 2011]

39. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $A + A^T$. [CBSE 2010]

40. If $\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$, then find a.

41. If A is a matrix of order 3×4 and B is a matrix of order 4×3, find the order of the matrix of AB. [CBSE 2010]

42. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ is identity matrix, then write the value of α. [CBSE 2010]

43. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then write the value of k. [CBSE 2010]

44. If I is the identity matrix and A is a square matrix such that $A^2 = A$, then what is the value of $(I + A)^2 - 3A$?

45. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is written as $B + C$, where B is a symmetric matrix and C is a skew-symmetric matrix, then find B.

46. If A is 2×3 matrix and B is a matrix such that $A^T B$ and BA^T both are defined, then what is the order of B?

47. What is the total number of 2×2 matrices with each entry 0 or 1?

48. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then find the value of y. [CBSE 2011]

49. If a matrix has 5 elements, write all possible orders it can have. [CBSE 2011]

50. For a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}. [CBSE 2011]

51. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the value of x. [CBSE 2012]

52. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find matrix A. [CBSE 2013]

53. If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the value of b. [CBSE 2013]

54. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix? [CBSE 2013]

DETERMINANTS **SECTION A**

1. For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ singular? [CBSE 2011]

2. A matrix A of order 3×3 is such that $|A| = 4$. Find the value of $|2A|$. [CBSE 2011]

3. Evaluate: $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$. [CBSE 2011]

4. If $A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$. Write the cofactor of the element a_{32}. [CBSE 2012]

5. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x. [CBSE 2013]

6. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then write the value of x. [CBSE 2013]

7. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x. [CBSE 2014]

8. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x. [CBSE 2014]

9. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$ then write the value of k. [CBSE 2014]

10. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$. [CBSE 2014]

11. Write the value of the determinant $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$. [CBSE 2015]

12. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number, find the value of Det (A^n). [CBSE 2015]

13. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ [CBSE 2016]

14. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x. [CBSE 2016]

15. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x. [CBSE 2016]

16. If A is a 3×3 matrix, then what will be the value of k if Det $(A^{-1}) = (\text{Det } A)^k$? [CBSE 2017]

17. A and B are square matrices of the same order 3, such that $AB = 2I$ and $|A| = 2$, write the value of $|B|$. [CBSE 2019]

18. A square matrix A is said to be singular, if ..... [CBSE 2020]

SECTION B

1. Let $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$ and $B=\begin{bmatrix}6 & 7\\ 8 & 9\end{bmatrix}$. Find $(AB)^{-1}$

2. Given $A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$, compute A^{-1} and show that $2A^{-1}=9I-A$. [CBSE 2018]

3. If $A=\begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix}$, then show that $A-3I=2(I+3A^{-1})$.

4. Find the inverse of the matrix $A=\begin{bmatrix}a & b\\ c & \dfrac{1+bc}{a}\end{bmatrix}$ and show that $aA^{-1}=(a^2+bc+1)I-aA$.

5. Given $A=\begin{bmatrix}5 & 0 & 4\\ 2 & 3 & 2\\ 1 & 2 & 1\end{bmatrix}$, $B^{-1}=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$. Compute $(AB)^{-1}$. [NCERT]

6. Let $F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$ and $G(\beta)=\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix}$. Show that

   (i) $[F(\alpha)]^{-1}=F(-\alpha)$ (ii) $[G(\beta)]^{-1}=G(-\beta)$ (iii) $[F(\alpha)G(\beta)]^{-1}=G(-\beta)F(-\alpha)$.

7. If $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$, verify that $A^2-4A+I=O$, where $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $O=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$. Hence, find A^{-1}. [NCERT]

8. Show that $A=\begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix}$ satisfies the equation $A^2+4A-42I=O$. Hence, find A^{-1}.

9. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$. Hence, find A^{-1}. [NCERT, CBSE 2007]

10. If $A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2-xA+yI=O$. Hence, evaluate A^{-1}.

11. If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$, find the value of λ so that $A^2=\lambda A-2I$. Hence, find A^{-1}. [CBSE 2007]

12. Show that $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ satisfies the equation $x^2-3x-7=0$. Thus, find A^{-1}.

13. Show that $A=\begin{bmatrix}6 & 5\\ 7 & 6\end{bmatrix}$ satisfies the equation $x^2-12x+1=0$. Thus, find A^{-1}

SECTION C

1. $A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$, find AB. Hence, solve the system of equations:

$x-2y=10,\ 2x+y+3z=8$ and $-2y+z=7$ **[CBSE 2011]**

2. If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of linear equations

$x-2y=10,\ 2x-y-z=8,\ -2y+z=7$ **[NCERT EXEMPLAR]**

3. Given $A=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$, find BA and use this to solve the system of equations $y+2z=7,\ x-y=3,\ 2x+3y+4z=17$ **[NCERT EXEMPLAR]**

4. If $A=\begin{bmatrix}2 & 3 & 1\\ 1 & 2 & 2\\ -3 & 1 & -1\end{bmatrix}$, find A^{-1} and hence solve the system of equations

$2x+y-3z=13,\ 3x+2y+z=4,\ x+2y-z=8.$ **[CBSE 2017]**

5. Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$x+3z=-9,\ -x+2y-2z=4,\ 2x-3y+4z=-3.$ **[CBSE 2017]**

6. The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the three numbers by using matrices.

7. An amount of ₹ 10,000 is put into three investments at the rate of 10, 12 and 15% per annum. The combined income is ₹ 1310 and the combined income of first and second investment is ₹ 190 short of the income from the third. Find the investment in each using matrix method.

8. A company produces three products every day. Their production on a certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product. Determine the production level of each product using matrix method.

9. The prices of three commodities P, Q and R are ₹ x, y and z per unit respectively. A purchases 4 units of R and sells 3 units of P and 5 units of Q. B purchases 3 units of Q and sells 2 units of P and 1 unit of R. C purchases 1 unit of P and sells 4 units of Q and 6 units of R. In the process A, B and C earn ₹ 6000, ₹ 5000 and ₹ 13000 respectively. If selling the units is positive earning and buying the units is negative earnings, find the price per unit of three commodities by using matrix method.

- **DO THE WORK IN SEPARATE NOTEBOOK.**