<u>11th Mathematics Holiday Home Work</u>

<u>SETS</u>

1. Decide, among the following sets, which are subsets of which.
   A = $\{x : x$ is a solution of $x^2 - 8x + 12 = 0\}$, B = $\{2, 4, 6\}$,
   C = $\{x : x$ is an even natural number$\}$, D = $\{6\}$.
2. State whether each of the following statements is true or false for the sets A and B where
   A = $\{x : x$ is a letter in the word TRACT$\}$,
   B = $\{x : x$ is a letter in the word CATARACT$\}$
   (*i*) $n(A) = 5$ (*ii*) $n(B) = 8$ (*iii*) $A \subset B$
   (*iv*) A is a proper subset of B (*v*) A = B.
3. Let ξ = the set of all letters in the word 'TAMILNADU' and
   X = $\{x : x$ is a vowel and $x \in \xi\}$
   (*i*) Write ξ and X in the roster form.
   (*ii*) Tell $n(\xi)$ and $n(X)$.
   (*iii*) List all the proper subsets of X.
   (*iv*) What is the cardinal number of the power set of X?
4. Let A be the set of letters in the word "POOR". Write the power set of A.
5. Find the power sets of the following sets:
   (*i*) $\{-1, 0, 1\}$ (*ii*) $\{0, 1, \{0, 1\}\}$.
6. If A = $\{2, 3, 5, 7, 8\}$, B = $\{1, 5, 9\}$ and A′ = $\{1, 4, 6, 9\}$, verify that
   (*i*) $(A \cup B)' = A' \cap B'$ (*ii*) $B - A = A' \cap B$.
7. For all sets A, B and C, is $A - (B - C) = (A - B) - C$ true? Justify your answer. (*Exemplar*)
8. If $n(\xi) = 30$, $n(A') = 15$, $n(B) = 5$ and $n(A \cap B) = 3$, find
   (*i*) $n(A)$ (*ii*) $n(A \cup B)$ (*iii*) $n(A - B)$.
9. If $n(\xi) = 40$, $n((A \cup B)') = 12$, $n(A - B) = 10$ and $n(B - A) = 14$, find
   (*i*) $n(A)$ (*ii*) $n(B)$ (*iii*) $n(A \cap B)$.
10. Two sets A and B are such that $n(A \cup B) = 18$, $n(A' \cap B) = 3$ and $n(A \cap B') = 5$, find $n(A \cap B)$.
11. Two sets A and B are such that $n(A \cup B) = 21$, $n(A' \cap B') = 9$ and $n(A \cap B) = 7$, find $n((A \cap B)')$.
12. If $n(\xi) = 50$, $n(A) = 3x$, $n(B) = 2x$ and $n(A \cap B) = x = n((A \cup B)')$, find
    (*i*) the value of x (*ii*) $n(A - B)$.
13. If $n(\xi) = 15$, A and B are two sets such that $A \subset B$, $n(A) = 8$ and $n(B) = 12$, use Venn diagram to find the following:
    (*i*) $n(A')$ (*ii*) $n(B')$ (*iii*) $n(A \cap B')$ (*iv*) $n(A' \cap B)$.

14. In a survey of 400 students in a school, 100 were listed as drinking coffee, 150 as drinking tea and 75 were listed both coffee as well as tea. Find how many students were drinking neither coffee nor tea.

15. In an examination, 56 percent of the candidates failed in English and 48 percent failed in Science. If 18 percent failed in both English and Science, find the percentage of those who passed in both the subjects.

16. From amongst the 6000 literate individuals of a city, 50% read newspaper A, 45% read newspaper B and 25% read neither A nor B. How many individuals read both the newspapers A as well as B?

17. In a beauty contest, half the number of judges voted for Miss A, $\frac{2}{3}$ of them voted for Miss B, 10 voted for both and 6 did not vote for either Miss A or Miss B. Find how many judges, in all, were present there.

18. In a group of 50 students, the number of students studying French, English and Sanskrit were found to be as follows:

French = 17, English = 13, Sanskrit = 15;

French and English = 9, English and Sanskrit = 4, French and Sanskrit = 5;

English, French and Sanskrit = 3.

Find the number of students who study:

(i) French only
(ii) French and Sanskrit but not English
(iii) English only
(iv) French and English but not Sanskrit
(v) Sanskrit only
(vi) English and Sanskrit but not French
(vii) atleast one of the three languages
(viii) none of the three languages. (*Exemplar*)

19. If A and B are two sets such that $n(A) = 10$ and $n(B) = 7$, then find:

(i) the least value of $n(A \cap B)$
(ii) the greatest value of $n(A \cap B)$
(iii) the greatest value of $n(A \cup B)$
(iv) the least value of $n(A \cup B)$.

RELATION & FUNCTION

1. If $A = \{1, 2, 3\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$, then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(iii) $A \times (B - C) = (A \times B) - (A \times C)$.

2. Let $A = \{2, 4, 6, 8\}$ and $B = \{0, 6, 8, 9, 10\}$. Find the elements of $(A \cap B) \times (A - B)$ corresponding to the relation '*is a multiple of*'.

3. Let $A = \{6, 7, 8, 10\}$, $B = \{2, 4, 5\}$, $a \in A$, $b \in B$ and R be the relation from A to B defined by $a\,R\,b$ if and only if a is divisible by b. Write R in the roster form.

4. Let $R = \{(x, y) ; x + 2y < 6, x, y \in N\}$

(i) Find the domain and the range of R
(ii) Write R as a set of ordered pairs.

5. Let $R = \{(x, y) ; y = x + 1 \text{ and } y \in \{0, 1, 2, 3, 4, 5\}\}$.

(i) List the elements of R.
(ii) Represent R by an arrow diagram.

6. Let f be the subset of $Q \times Z$ defined by $f = \left\{\left(\frac{m}{n}, m\right) : m, n \in Z, n \neq 0\right\}$. Is f a function from Q to Z? Justify your answer.

7. Let $f : X \to Y$ be defined by $f(x) = x^2$ for all $x \in X$ where $X = \{-2, -1, 0, 1, 2, 3\}$ and $Y = \{0, 1, 4, 7, 9, 10\}$.
Write the relation f in the roster form. Is f a function?

8. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If this is described by the relation $g(x) = \alpha x + \beta$, then what values should be assigned to α and β. *(Exemplar)*

9. Determine a quadratic function 'f' defined by
$$f(x) = ax^2 + bx + c \text{ if } f(0) = 6, f(2) = 11 \text{ and } f(-3) = 6.$$

10. Find the domain and the range of the function $f(x) = 2 - 3x^2$. Also find $f(-2)$ and the numbers which are associated with the number -25 in its range.

11. Find the domain and the range of the following functions:
   (i) $\sqrt{x-3}$ (ii) $\sqrt{25 - x^2}$ (iii) $5 - |x + 1|$.

12. Draw the graph of the function $f(x) = \begin{cases} 1 + 2x, & x < 0 \\ 3 + 5x, & x \geq 0 \end{cases}$.
Hence, find its range.

13. If $f(x) = 2x + 5$ and $g(x) = x^2 - 1$ are two real valued functions, find the following functions:
   (i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$ (v) $\frac{g}{f}$ (vi) $3g + 2f^2$.

TRIGONOMETRY

1. Find the radian measure of an angle (internal) of a regular
   (i) pentagon (ii) hexagon (iii) polygon of n sides.

2. Find the value of $\sin\left(-\frac{41\pi}{4}\right)$.

3. Prove that:
   (i) $\sin^2\frac{\pi}{6}, \sin^2\frac{\pi}{4}, \sin^2\frac{\pi}{3}$ are in A.P. (ii) $\tan^2\frac{\pi}{6}, \tan^2\frac{\pi}{4}, \tan^2\frac{\pi}{3}$ are in G.P.

4. Find the value of $m^2 \sin\frac{1}{2}\pi - n^2 \sin\frac{3}{2}\pi + 2mn \sec \pi$.

5. What is the maximum value of $3 - 7\cos 5x$?

6. What is the minimum value of $4 + 5\sin(3x - 2)$?

7. What is the maximum value of $\sin x \cos x$?

8. What is the maximum value of $3\sin x - 4\sin^3 x$?

9. What is the minimum value of $3\cos x - 4\cos^3 x$?

10. What is the least value of $2\sin^2 x + 3\cos^2 x$?

11. What is the maximum value of $\sin x + \cos x$?

12. What is the minimum value of $\sin x - \cos x$?

13. If $\sin 2x = \cos 3x$ and $0 \leq x < \frac{\pi}{2}$, then find the value of x.

14. A railway carriage is travelling along a circular railway track of radius 1500 metres with a speed of 66 km / hour. Find the angle in degrees turned by the engine in 10 seconds.

15. If $\tan x = \frac{a}{b}$, show that $\frac{a \sin x - b \cos x}{a \sin x + b \cos x} = \frac{a^2 - b^2}{a^2 + b^2}$.

16. Is the equation $6 \sec^2 x - 5 \sec x + 1 = 0$ possible?

17. Show that $\sqrt{3}\ (\tan 170° - \tan 140°) = 1 + \tan 170° \tan 140°$.

18. If $A + B = 45°$, prove that $(1 + \tan A)(1 + \tan B) = 2$.

Hence, find the value of $\tan 22\frac{1}{2}^{\circ}$.

19. If $\tan y = \frac{Q \sin x}{P + Q \cos x}$, prove that $\tan (x - y) = \frac{P \sin x}{Q + P \cos x}$.

20. Prove that $\cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2 x + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$.

21. If x, y and z are in A.P., prove that $\cot y = \frac{\sin x - \sin z}{\cos z - \cos x}$.

22. If $\sin 2x = \lambda \sin 2y$, prove that $\frac{\tan (x + y)}{\tan (x - y)} = \frac{\lambda + 1}{\lambda - 1}$.

23. Prove that $\frac{1 - \sin 2x}{1 + \sin 2x} = \tan^2\left(\frac{\pi}{4} - x\right)$.

24. Prove that $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$.

25. Given that $\sin x = -\frac{3}{5}$, $\cos y = -\frac{5}{13}$ and x is in the same quadrant as y, evaluate:

(i) $\cos (x - y)$ (ii) $\tan (x + y)$ (iii) $\cos \frac{x}{2}$.

26. Prove that:

(i) $\dfrac{1-\cos x+\cos y-\cos(x+y)}{1+\cos x-\cos y-\cos(x+y)}=\tan\dfrac{x}{2}\cot\dfrac{y}{2}$

(ii) $\dfrac{\cos^3 x-\cos 3x}{\cos x}+\dfrac{\sin^3 x+\sin 3x}{\sin x}=3.$

27. Prove that $\dfrac{1}{\sin 10°}-\dfrac{\sqrt{3}}{\cos 10°}=4.$

28. Show that $\dfrac{\tan 3x}{\tan x}$ never lies between $\dfrac{1}{3}$ and 3.

29. Solve the following equations:

(i) $\tan 2x=-\cot\left(x+\dfrac{\pi}{6}\right)$

(ii) $\cot^2 x+3\operatorname{cosec} x+3=0$

(iii) $4\sin^2 x+\sqrt{3}=2(1+\sqrt{3})\sin x$

(iv) $\tan^2 x-(1+\sqrt{3})\tan x+\sqrt{3}=0$

(v) $\cos 2x-\cos 8x+\cos 6x=1$

(vi) $\tan\left(\dfrac{\pi}{4}+x\right)+\tan\left(\dfrac{\pi}{4}-x\right)=4$

(vii) $\operatorname{cosec} x=1+\cot x.$

- **DO THE WORK IN SEPARATE NOTEBOOK.**